ELSEVIER

Human Movement Science 18 (1999) 769–793

HUMAN MOVEMENT SCIENCE

www.elsevier.com/locate/humov

Effects of practice and task constraints on stiffness and friction functions in biological movements

D. Delignières *, D. Nourrit, T. Deschamps, B. Lauriot, N. Caillou

E.A. "Sport – Performance – Santé", Faculty of Sport and Physical Education Sciences, University of Montpellier I, 700 Avenue du Pic Saint Loup, F-34090 Montpellier, France

Abstract

A dynamical model of the movements of the platform of a ski-simulator was derived from experimental data, using the graphical and statistical methods developed by Beek and Beek (Beek, P. J., & Beek, W. J. (1988). Human Movement Science, 7, 301–342). The data were collected in an experiment in which both amplitude and practice were manipulated. The data were filtered and further reduced to normalised cycles that were averaged within and across subjects. Graphical methods were applied to these averaged normalised cycles to determine the stiffness and friction terms to be included in the model. The relative contribution of each term was assessed by means of multiple regression. The model, which included cubic and quintic Duffing terms and one or two dissipative Van der Pol terms, accounted on average for 99.2% of the variance. The exact parameter setting of the model differed considerably across subjects. For one subject, a qualitatively different model, including Rayleigh instead of Van der Pol terms, provided a better account of the data. Systematic changes of the coefficients in the model, related to amplitude and the duration of practice, were evident. © 1999 Elsevier Science B.V. All rights reserved.

PsycINFO classification: 2330

Keywords: Dynamical model; Non-linear stiffness; Non-linear friction

* Corresponding author. Tel.: +33-04-67-41-57-54; fax: +33-04-67-41-57-08.
E-mail address: delignieres@sc.univ-montp1.fr (D. Delignières).

PII: S 0 1 6 7 - 9 4 5 7 (9 9) 0 0 0 4 0 - 8

1. Introduction

Recently, several attempts have been made to model biological rhythmic movements as self-sustained oscillators (e.g., Haken, Kelso & Bunz, 1985; Kay, Saltzman, Kelso & Schöner, 1987; Beek & Beek, 1988; Beek, Schmidt, Morris, Sim & Turvey, 1995b; Beek, Rikkert & van Wieringen, 1996). These attempts are based on the assumption that the central nervous system employs limit-cycle dynamics to produce rhythmic movements. Their aim is to provide macroscopic models that contain a small number of parameters [1] and that capture the essential features of rhythmic movement (i.e., the average attractive dynamic pattern). The resulting dynamical equations of motion express at an abstract level the interplay of the physiological properties of the supporting neural structures and the (bio)mechanical properties of the oscillating system. Theoretically, this notion is supported by the idea that macroscopic order arises out of the non-linear interactions between microscopic elements but in turns governs the functioning of these elements (Haken, 1983; Beek, Peper & Stegeman, 1995a).

In this framework, rhythmic movements are modelled as oscillators obeying second-order ordinary differential equations of the kind

$$m\ddot{x} + f(x,\dot{x})\dot{x} + g(x) = 0, \quad (1)$$

where x represents position. The dot notation indicates differentiation with respect to time. The first term expresses the inertia of the system, the second the system's friction (or damping), and the third the system's stiffness. The major concern of this approach is to identify the non-linear stiffness (elasticity) and damping (escapement) functions that are exploited to produce rhythmic movement.

Beek and Beek (1988) showed that the stiffness and friction terms composing non-linear oscillations may be represented as a series of terms $x^p\dot{x}^q$ $(p,q : 0,1,2,3,\ldots)$, and that only a limited number of such terms represent viable transformations of the harmonic oscillator $(\ddot{x} + x = 0)$. The aim of these authors was to account for the successive phases of the hand motion in juggling in terms of the quantitative dynamics of a single autonomous oscillator. Developing a model by means of Chebychev-type polynomials,

[1] This minimality criterion is necessary for a valid assessment of the coefficients in the model using multiple regression procedures (see Section 2).

they showed that the continuous representation of such a biological motion imposes restrictions on the kind of terms to be included. More specifically, $g(x)$ should be composed of terms from the Duffing series $(x^1, x^3, x^5, \ldots)$, and $f(x, \dot{x})$ should be composed of terms from the Van der Pol series $(x^0, x^2, x^4, \ldots)$ and/or from the Rayleigh series $(\dot{x}^0, \dot{x}^2, \dot{x}^4, \ldots)$, either separately or in combination. They also showed the viability, in addition to the well-known non-linearities of Rayleigh and Van der Pol, of a new type of series expansions that they called π-mix series (even terms: $x^2\dot{x}^2, x^4\dot{x}^4, \ldots$; odd terms: $x^3\dot{x}^1, x^3\dot{x}^4, \ldots$). Recent experiments have demonstrated that the catalogue of Beek and Beek (1988) allows for the construction of models that adequately capture the main kinematic properties of various kinds of rhythmic and discrete movements (e.g., Beek et al., 1995a,b, 1996; Mottet & Bootsma, 1999; Zaal, Bootsma & van Wieringen, 1998).

Numerous methods are available to determine the nature of the stiffness and damping functions and to determine the relative importance of each term. One method was pursued by Kay et al. (1987) in modelling the dynamics of single and bimanual rhythmical movements. On the assumption that the stiffness function was linear, the authors analysed the frequency–amplitude and frequency–peak velocity relations of (paced) rhythmic hand movements to derive the components of the damping function. Following this analysis, they proposed a so-called "hybrid model", which involves a combination of the non-linear Rayleigh and Van der Pol terms. Using the same method, Beek et al. (1996) showed for rhythmic forearm movements that a modified version, including a frequency-dependent Rayleigh term, provided a better account of the individual data. As these two experiments involved different anatomical oscillators (wrist movements and elbow movements, respectively), the authors explained the discrepancy between their modelling results and those obtained by Kay et al. (1987) by differences in biomechanical properties, such as moment of inertia, joint stiffness and damping.

Beek and Beek (1988) proposed a graphical method that may be used to reveal the presence of specific non-linear terms in the stiffness and damping functions. Exploiting the presence, in these terms, of easily recognisable elements (such as x^2, x^3 or x^4), the authors showed that specific graphical representations of the kinematic data allowed to scout for local expressions of non-linear terms. This method, applied to the analysis of the kinematics of hand movement of juggling, revealed the presence of local Duffing and Van der Pol-behaviours. This method will be explained in more detail in Section 2.

An additional statistical procedure (the so-called W-method) was also developed by Beek and Beek (1988); see also Beek et al. (1995b). Their starting point was a specific version of Eq. (1):

$$\ddot{x} + x + W(x, \dot{x}) = 0, \tag{2}$$

where $W(x, \dot{x})$ summarises the contribution of all linear and non-linear conservative and dissipative components of the motion besides the inertial force and the restoring force (i.e., the continuous deviation of oscillatory motion from ideal harmonic motion). $W(x, \dot{x})$ can be easily computed from kinematic data, and is used as a dependent variable on which permissible terms are regressed to determine which coefficients contribute significantly to its variation in time. Beek et al. (1995b) used this method to determine the contribution of non-linear stiffness and friction terms in pendulum swinging. This procedure allowed them to propose a model including a cubic Duffing term, a π-mix term of the form $x\dot{x}^2$, and a combination of friction terms suggestive of a Van der Pol oscillator ($\dot{x}$, $x^2\dot{x}$). The authors described systematic changes of model coefficients, under manipulations of rotational inertia, frequency and amplitude.

Using an adapted version of the W-method (see later), Mottet and Bootsma (1999) showed that, in a reciprocal Fitts' task, the end-effector dynamics could be adequately modelled with a limit-cycle model involving a non-linear stiffness in the form of a Duffing term (x^3) and a non-linear damping in the form of a Rayleigh term ($\dot{x}^3$). The model reproduced the main features of the kinematic data and accounted for 95% of the variance. Mottet and Bootsma (1999) showed that the coefficients in the model changed in a systematic fashion when distance and precision constraints were varied.

Beek et al. (1995b) noted that the W-method is better suited for estimating the stiffness (conservative) terms than for estimating the damping (non-conservative) terms, because, on average, the effects of the various damping terms are cancelled out along the limit-cycle (i.e., averaged over the limit-cycle energy is neither lost nor gained). Furthermore, the method can suffer from the inability of the regression procedure to take into account the sign constraints that are needed to obtain self-sustaining models. To give rise to a limit-cycle, the linear damping term must be negative, and at least one of the non-linear damping terms must be positive. Mottet and Bootsma (1999) reported that stepwise regression of all possible terms in $W(x, \dot{x})$ mostly led to inconsistent results (e.g., unstable models, or non-significant linear damping).

Hence, the use of graphical methods to qualitatively derive a "minimal" model appears to be a good preliminary step in applying the W-method.

The aim of the present study was to generate a dynamical model of the motion of the platform of the ski-simulator, and to analyse the changes in the coefficients of the model with practice and under various amplitude constraints. The ski-simulator has been used in many experiments, especially in the domain of skill acquisition (den Brinker & van Hekken, 1982; den Brinker, Stäbler, Whiting & van Wieringen, 1986; van Emmerik, den Brinker, Vereijken & Whiting, 1989; Vereijken & Whiting, 1990; Vereijken, 1991; Vereijken, Whiting & Beek, 1992a; Vereijken, van Emmerik, Whiting & Newell, 1992b; Durand, Geoffroi, Varray & Préfaut, 1994; Vereijken, van Emmerik, Bongaardt, Beek & Newell, 1997; Wulf & Weigelt, 1997; Wulf, Höß & Prinz, 1998). In the present work, we considered the platform as an end-effector, assuming that its kinematics contains information about the overall co-ordination dynamics (Vereijken, 1991; Vereijken et al., 1992a).

Our first goal was to analyse the effects of practice on the platform dynamics. Vereijken et al. (1997) suggested that learning to ski on a ski-simulator proceeds in three stages involving qualitatively different co-ordinative structures, which they interpreted as different instantiations of pendulum systems (balancing pendulum, hanging pendulum and buckling compound pendulum). We hypothesised that such qualitative changes in global behaviour should lead to qualitative as well as quantitative changes in the composition of the dynamical model of the movement of the platform.

Our second goal was to study the effects of movement amplitude. This choice was motivated by the results of an earlier experiment (Delignières, Nourrit, Lauriot & Cadjee, 1997), which revealed the existence of a significant decrease in frequency variability as required amplitude increased. Considering that an increase in amplitude on the ski-simulator generally leads to an increase in the average velocity of the platform, this effect can be seen as an illustration of the relationship described by Newell, (1980) between average velocity and timing error. In dynamical terms, this decrease in frequency variability suggests an enhancement of the attractive power of the limit-cycle, which should be revealed by specific changes in the coefficients of the model (especially in the damping function, which plays an essential role in the attractiveness of the limit-cycle; cf. Mottet & Bootsma, 1999).

In summary, our goal was to analyse the effects on the dynamical model of practice and required amplitude, considered as independent factors. Thus, in the present experiment, practice is geared toward skill optimisation at a given amplitude.

2. Modelling strategy

The modelling strategy used in the present study is based on the analysis of an averaged normalised cycle, which is assumed to represent the dynamical organisation that emerged in response to the task demands. We suppose that the attractor does not change at the time scale of observation (i.e., within a single trial), and that the random fluctuations at small time and length scales are the result of random forces (noise) arising from the system's components, pushing the observed behaviour around the average attractive pattern.

Our method combines qualitative graphical analyses, to identify the non-linear components describing platform movements, and quantitative statistical procedures, to estimate the magnitude of the respective contribution of each component, and their change with practice and required amplitude.

First we used a Hooke's plane representation (position vs. acceleration) in order to directly assess the stiffness function (Guiard, 1993; Mottet & Bootsma, 1999). For a perfectly harmonic oscillator Hooke's portrait shows a straight line. Deviations from this straight line provide information about the non-linear stiffness terms that should be included in the model. Some typical Hooke's portraits are shown in Fig. 1, with (top) the straight line obtained for a harmonic oscillator ($\ddot{x} + x = 0$), and (middle) the N-shape revealing the presence of a negative Duffing cubic term ($\ddot{x} + x - x^3 = 0$). The bottom chart was obtained by the inclusion, in the previous equation, of a positive quintic Duffing term ($\ddot{x} + x - x^3 + x^5 = 0$).

The determination of relevant non-linear damping terms is less straightforward than that of the stiffness terms. One method is to use stepwise regression of all viable terms (i.e., $x, x^3, x^5, \dot{x}, \dot{x}^3, x^2\dot{x}$, see Beek & Beek, 1988). However, as previously indicated, the regression procedure is unable to take into account the sign constraints on the damping coefficients that need to be satisfied to generate limit-cycle dynamics. We therefore used graphical analyses to determine the relevant non-linear damping terms. To isolate the contribution of non-linear damping, we first regressed all previously identified linear and non-linear stiffness terms and linear damping ($\dot{x}$) on $-\ddot{x}$. The residual (RES) of this regression was assumed to reflect the contribution of the non-linear damping terms on behaviour. Then we applied the principles proposed by Beek and Beek (1988), for example to scout for local Van der Pol or Rayleigh behaviour, respectively by plotting the value of RES/$\dot{x}$ as a function of x, and by plotting the value of RES as a function of $\dot{x}$.

The aim of this graphical analysis was to determine a minimal dynamical model, containing a limited set of relevant terms. Subsequently,

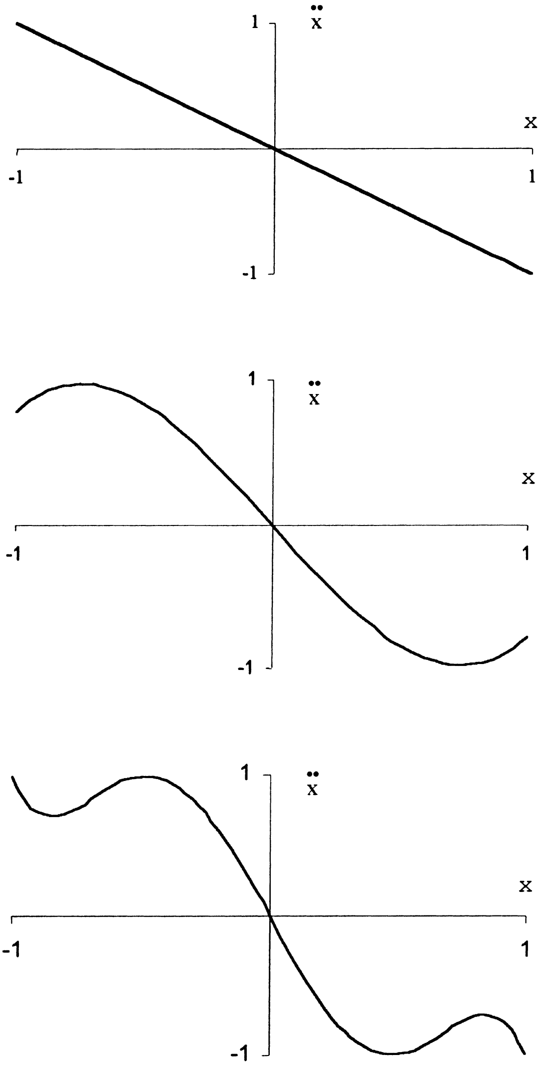

Fig. 1. Hooke's plane representations (position vs. acceleration). The top chart represents a perfect harmonic oscillator. The two other charts illustrates the influence of cubic (middle) and quintic (bottom) Duffing stiffness terms. Equations: $\ddot{x} + x = 0$, $\ddot{x} + x - 0.6x^3 = 0$ and $\ddot{x} + x - 1.9x^3 + 1.2x^5 = 0$, respectively. For presentation convenience, acceleration data were rescaled within the interval [−1,+1].

the relative importance of each coefficient was assessed by a multiple regression procedure, as suggested by the original W-method (Beek & Beek, 1988).

3. Method

3.1. Subjects

Fifteen subjects (mean age: 23.8 years ± 2.3, mean weight: 71.0 kg ± 6.5, mean height 176.8 cm ± 6.8) volunteered for the experiment. All were occasional skiers, but none of them had previous experience on the ski-simulator. They signed a consent form, and were not paid for their participation.

3.2. Task

The task was performed on a slalom ski-simulator (Skier's Edge, Fig. 2), consisting of a platform on wheels which could move back and forth on two bowed, parallel metal rails. The subject's feet were strapped to the platform, which in turn was fastened to the rails by means of two adjustable rubber belts. The tension of the belts was controlled with a dynamometer at the beginning of each session, and adjusted so as to obtain a displacement of 4 cm of the platform from the central position with a tangential force of 100 N.

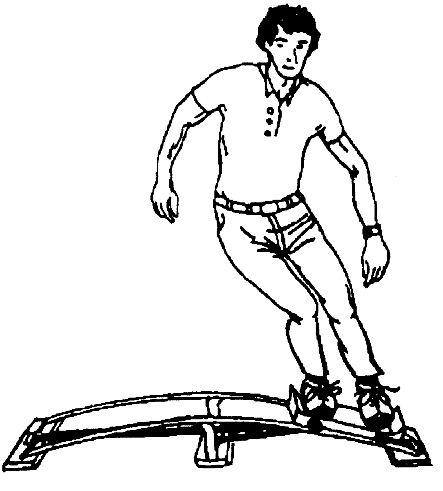

Fig. 2. The ski-simulator apparatus (see text for details). From Whiting, Vogt and Vereijken (1992).

3.3. Procedure

The subjects were randomly assigned to the three experimental groups. They were asked to perform slalom-like movements on the apparatus with an amplitude (measured from the midpoint to the left or right reversal point) of 30 cm (group A), 22.5 cm (group B) or 15 cm (group C). Two fibreglass sticks were adjusted vertically on both sides of the apparatus to indicate the target amplitude. Note that the maximum possible amplitude of movement permitted by the apparatus was circa 50 cm. The selected target amplitudes were kept deliberately small since it was deemed desirable that each subject would have the same amount of practice at the target amplitude. Therefore, it was necessary that the required amplitude could be achieved from the very first trials of the experiment onwards. Moreover, a preliminary experiment had shown that these amplitudes were sufficient to induce significant between group differences in frequency variability (Delignières et al., 1997). Finally, these target amplitudes were sufficiently low to avoid wear and tear of the rubber belts.

Four practice sessions were conducted on four consecutive days. Each session consisted of four 4-min trials, with a 4-min break between them. Subjects were given no demonstration, but were told to perform the task as comfortably as possible (i.e., at their preferred frequency).

3.4. Data reduction

The position of the midpoint of the platform was measured by a potentiometer and sampled at a frequency of 100 Hz. Data were stored on a personal computer for further analyses.

For the purpose of the present study, the analyses were confined to the first 15 s of the third minute of the first trial of each session. The cumulative amount of practice, before each selected sample, was then 2, 18, 32 and 50 min, respectively. The extracted position time series were filtered with a dual-pass second-order Butterworth filter with a cut-off frequency of 10 Hz. This cut-off frequency was chosen following an analysis of the spectral composition of the time series, and was deemed appropriate because it preserved the essential characteristics of the signal.

A peak-finding algorithm was used to locate the reversal points of the movement. Cycle frequency (in hertzs) was defined as the inverse of the time between two successive right reversals. Cycle amplitude (in centimetres) was defined as the mean of the maximal deviations of the platform from the rest position, at the right and left reversal points of the cycle. Means and standard deviations of amplitude and frequency were computed for each 15-s sample.

Each sample was then summarised in an averaged normalised cycle, which was calculated as follows. Firstly, the 15-s time series were segmented into half-cycles representing the motion from one reversal point to the next. Each half-cycle was then normalised using 21 equidistant points by means of linear interpolation. These points were then rescaled to the interval $[-1, +1]$. The normalised half-cycles beginning at the same reversal point were averaged point by point, and the averaged normalised cycle (42 points) was constructed by combining the back and forth average normalised half-cycles. The first and second derivatives were computed from the averaged normalised cycle, and then rescaled to the interval $[-1, +1]$. As can be seen, our data were normalised both with respect to time (to enable the averaging of cycles), and with respect to amplitude. The normalisation of amplitude, for displacement, as well as for velocity and acceleration data, was done to facilitate inter-group comparisons in graphical analyses (see next session, and Fig. 3).

Each 15-s sample allowed the calculation of a minimum of 10 and a maximum of 18 complete normalised cycles depending on the frequency of the movement. To assess the consistency of the platform dynamics within the 15-s intervals, we calculated the correlation between each normalized cycle and the corresponding average normalized cycle. In all cases the coefficient of correlation was close to 1 (mean 0.998), indicating the stability of platform dynamics, for each subject and within each 15-s interval.

Finally, 12 group average cycles (3 amplitudes $\times$ 4 sessions) were computed by point-by-point averaging of the corresponding individual normalised cycles. Fig. 3 shows the Hooke's portraits (acceleration against position) obtained for each condition.

4. Results

4.1. Amplitude and frequency

As expected, a 3 (amplitude) $\times$ 4 (session) ANOVA revealed a significant effect of required amplitude on actual amplitude ($F_{2,12} = 64.857$, $P < 0.001$). Post-hoc tests showed that each group differed significantly from each other (group A = 31.94 cm, group B = 24.97 cm, group C = 17.12 cm). There was no significant effect of practice (indicating that all subjects were able to reach the required amplitudes from the beginning of the experiment), nor a sig-

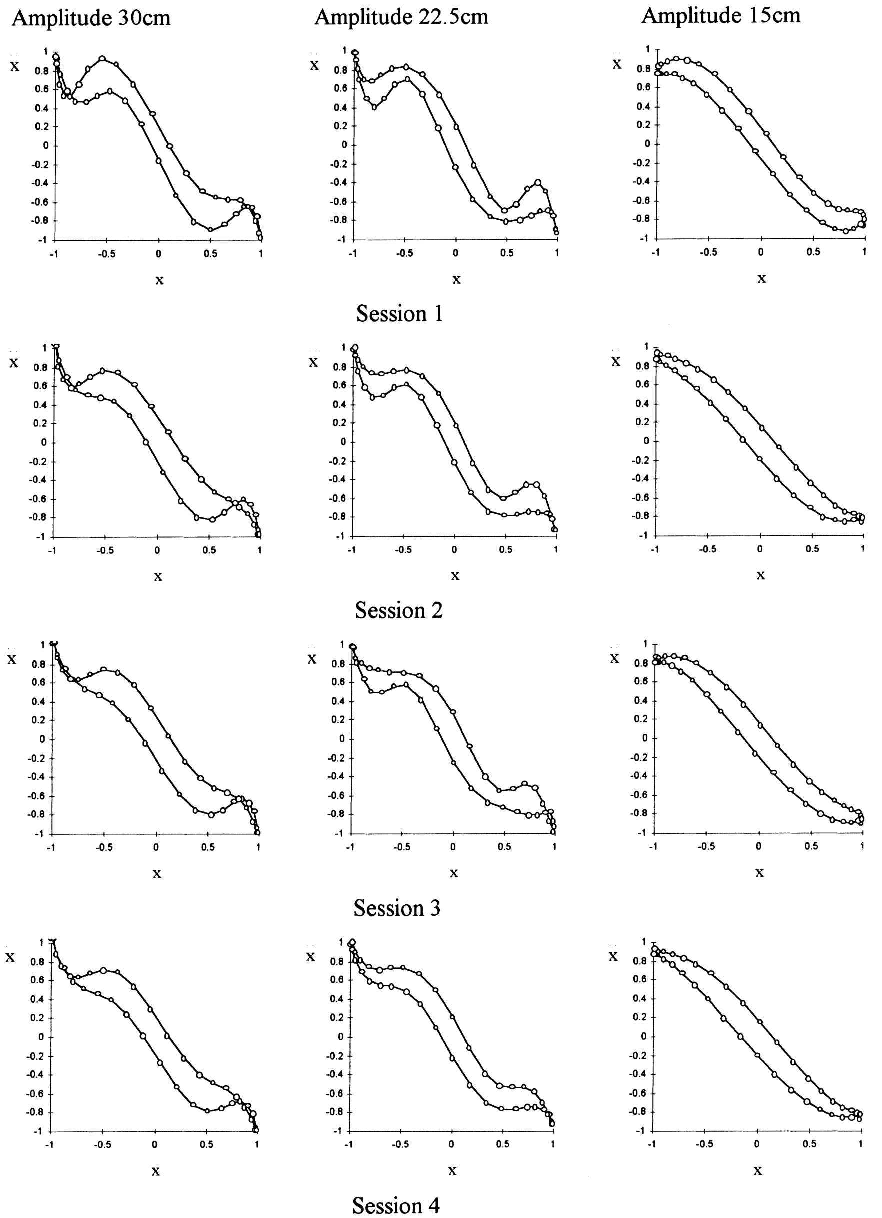

Fig. 3. Average normalised Hooke's portrait (acceleration against position), according to required amplitude (left to right), and practice session (top to bottom).

nificant interaction between amplitude and session. Concerning frequency, a significant effect of amplitude was revealed ($F_{2,12} = 10.255,\ P < 0.01$). Pairwise comparisons indicated that frequency was significantly higher for group C than for the two other groups (group A = 0.91 Hz, group B = 0.79 Hz, group C = 1.14 Hz). There was also a significant effect of practice ($F_{3,36} = 3.873,\ P < 0.05$). Post-hoc tests indicated that frequency was significantly lower during the first session, but remain unchanged during the remaining 3 sessions. No significant interaction effect was obtained.

As expected from a previous experiment (Delignières et al., 1997), we obtained a significant effect of target amplitude on frequency variability ($F_{2,12} = 15.516,\ P < 0.001$). Post-hoc comparisons revealed that mean frequency variability was significantly larger for the group performing the low amplitude movements (group C) than for groups B and A (0.031 vs. 0.019 and 0.023, respectively). There was no significant effect of practice, nor a significant interaction. The same analysis was performed on amplitude variability, but this analysis revealed no significant (main or interaction) effects.

4.2. Data modelling

From the r^2 of the linear regression of position onto acceleration, which is a measure of the variance that can be attributed to simple harmonic motion, it was possible to assess the relative contribution of the non-linear terms by the quantity $1 - r^2$ (Mottet & Bootsma, 1999). A 3 (amplitude)×4 (session) ANOVA revealed a significant effect of amplitude ($1 - r^2$ mean values: group A – 0.085, group B – 0.090, group C – 0.045; $F_{2,12} = 9.406,\ P < 0.005$). Post-hoc tests showed that the contribution of non-linear terms was significantly higher in groups A and B, than in group C. A significant session effect was also obtained (mean values: session 1 – 0.105, session 2 – 0.068, session 3 – 0.064, session 4 – 0.058; $F_{3,36} = 22.801,\ P < 0.001$), with significant linear and quadratic trends ($F_{1,12} = 27.061,\ P < 0.001$; and $F_{1,12} = 28.528,\ P < 0.001$, respectively). This effect suggests a gradual, but asymptotic decrease of the contribution of non-linear terms with practice. Finally, there was no significant interaction between practice and amplitude.

Visual inspection of the Hooke's portraits (see Fig. 3) indicated, especially for the 30 and 22.5 cm amplitude conditions, that the local stiffness tended to decrease near the reversal point and to increase again at the reversal point. This suggested that a negative cubic (x^3) and a positive quintic (x^5) Duffing term had to be included in the stiffness function of the equation of motion:

$$g(x) = x - x^3 + x^5.$$

The non-conservative damping terms were determined indirectly. As indicated in Section 1, we used a graphical method adapted from Beek and Beek (1988). First we computed the residuals (RES) of the regression of x, x^3, x^5 and $\dot{x}$ onto $-\ddot{x}$. Then we searched for Van der Pol-behaviour by plotting the value of RES/$\dot{x}$ as a function of x (in this case a parabola is expected) and for Rayleigh-behaviour by plotting the value of RES as a function of $\dot{x}$ (expecting a N-shape). A specific scouting the π-mix behaviour was also performed by plotting RES/$\dot{x}$ against $x\dot{x}$ (looking for parabola for π-mix even behaviour, and for N-shape for π-mix odd behaviour, see Beek and Beek (1988)).

In 9 cases out of 12 these graphical analyses revealed for the group average cycles a local Van der Pol-behaviour (Fig. 4, left panel), suggesting a general equation of motion reading

$$\ddot{x} + c_{10}x + c_{30}x^3 + c_{50}x^5 + c_{01}\dot{x} + c_{21}x^2\dot{x} = 0. \quad (3)$$

In this equation, the coefficients are indexed using the W-method notation proposed by Beek and Beek (1988), where c_{ij} denotes the coefficient of $x^i\dot{x}^j$.

On some occasions, nevertheless, two well-distinguished minima appeared when RES/$\dot{x}$ was plotted against x (see Fig. 4, right panel), suggesting that a quadratic Van der Pol should be added to the model:

$$\ddot{x} + c_{10}x + c_{30}x^3 + c_{50}x^5 + c_{01}\dot{x} + c_{21}x^2\dot{x} + c_{41}x^4\dot{x} = 0. \quad (4)$$

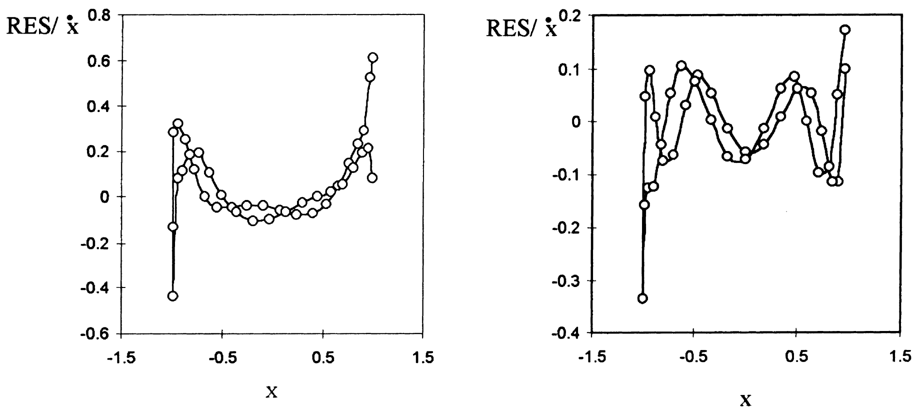

Fig. 4. Graphical scouting for Van der Pol-behaviour. Left panel: data from group A, session 1; right panel: data from group B, session 1.

At this level of analysis, no evidence for Rayleigh or π-mix terms was found.

The shape of the Hooke portraits suggests that c_{30} is negative and c_{50} positive. Furthermore, to give rise to limit-cycle dynamics, c_{01} should be negative and c_{21} positive in Eq. (3), and c_{21} and/or c_{41} positive in Eq. (4).

These coefficients were estimated using a stepwise multiple regression procedure of all relevant terms x, x^3, $x^5\dot{x}$ and $x^2\dot{x}$ (plus eventually $x^4\dot{x}$ in cases in which this term appeared necessary) onto $-\ddot{x}$. These regressions revealed that Eqs. (3) and (4) accurately predicted the observed behaviour with r^2 ranging from 0.992 to 1.000 (mean 0.997). The best fits where obtained for the data from group C (15 cm). As can be seen in Table 1, the estimated coefficients of the stiffness terms had the expected signs, c_{10} and c_{50} being positive, and c_{30} negative. Moreover, the sign requirements for the linear and non-linear damping terms were met in all cases, with negative values for c_{01}, positive for c_{21}, and negative, when present, for c_{41}.

Visual inspection of the estimated coefficients in Table 1 suggests the presence of a strong effect of movement amplitude on the stiffness components: considering absolute values, c_{10}, c_{30} and c_{50} seem lower for group C than for the two other groups, with no apparent differences between groups A and B. An effect of amplitude on linear friction seems plausible, with higher estimates (again in absolute values) for group A than for groups B and C. Finally, c_{21} seems lower for group C than for group A. Table 1 also suggests a systematic effect of practice on the magnitude of the stiffness co-

Table 1
Estimates of the stiffness and damping coefficients for the averaged normalised cycles

Group	Session	c_{10}	c_{30}	c_{50}	c_{01}	c_{21}	c_{41}	c_{03}	r^2
A	1	2.168	−2.002	0.845	−0.221	0.091	–	–	0.993
	2	1.833	−1.516	0.699	−0.253	0.141	–	–	0.997
	3	1.686	−1.285	0.621	−0.240	0.124	–	–	0.998
	4	1.548	−1.069	0.545	−0.229	0.108	–	–	0.997
B	1	2.135	−1.971	0.871	−0.185	0.103	−0.085	–	0.996
	2	1.950	−1.700	0.792	−0.189	0.113	−0.088	–	0.996
	3	1.795	−1.410	0.643	−0.166	0.016[a]	–	–	0.992
	4	1.720	−1.250	0.550	−0.196	0.059	–	–	0.998
C	1	1.543	−0.609	0.033	−0.158	0.028	−0.037	–	1.000
	2	1.349	−0.380	0.003[a]	−0.173	0.025	–	–	0.999
	3	1.381	−0.424	0.016	−0.171	0.023	–	–	1.000
	4	1.352	−0.427	0.055	−0.182	0.039	–	–	0.999

[a] Not significantly different from zero on a t test.

efficients, with a progressive decrease (in absolute values) of c_{10}, c_{30} and c_{50} estimates. There was no clear trend concerning the coefficients of the damping terms. These results provide a first insight into the respective effects of amplitude and practice on the stiffness and friction functions, but this insight has to be confirmed by statistical procedures involving individual data.

4.3. Individual modelling

The same graphical and statistical procedures were applied to individual normalised cycles. For 37 individual cycles (62%), Eq. (3) provided the most relevant model, whereas Eq. (4) provided the best description of the data for 18 other individual cycles (30%).

For one subject of group C, the plot of RES as a function of $\dot{x}$ revealed a typical Rayleigh behaviour (see Fig. 5). Conversely, the plot of RES/$\dot{x}$ as a function of x failed to evidence any kind of Van der Pol-behaviour. Thus, for this subject Eq. (4) was used for estimating the coefficients. Note that for one trial of one subject of group B (see Table 3), this combination of Duffing + Rayleigh behaviour also appeared to provide the most appropriate fit of the data.

These models provided an appropriate fitting of experimental data, with r^2 ranging from 0.955 to 1.000, with a mean of 99.2% of explained variance. As can be seen in Tables 2–4, the estimated values for stiffness terms c_{10} and c_{30} had the expected signs (positive for the first, negative for the second). The

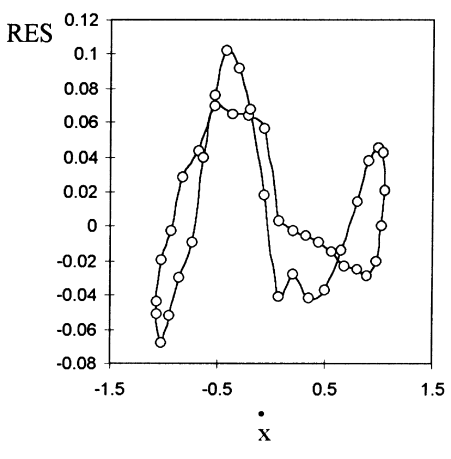

Fig. 5. Graphical scouting for Rayleigh behaviour. Data from subject C2, session 1.

Table 2
Estimates of the stiffness and damping coefficients for the individual normalised cycles of group A

Group	Session	c_{10}	c_{30}	c_{50}	c_{01}	c_{21}	c_{41}	c_{03}	r^2
A1	1	2.558	−2.239	0.603	−0.162	−0.007[a]	–	–	0.977
	2	1.961	−1.695	0.765	−0.184	0.040	–	–	0.997
	3	1.841	−1.472	0.656	−0.192	0.052	–	–	0.996
	4	1.102	−0.125	0.013[a]	−0.182	0.040	–	–	0.998
A2	1	1.875	−1.680	0.785	−0.344	0.272	–	–	0.973
	2	1.457	−0.824	0.323	−0.367	0.305	–	–	0.997
	3	1.138	−0.301	0.122	−0.344	0.272	–	–	0.999
	4	1.733	−1.412	0.690	−0.278	0.179	–	–	0.991
A3	1	2.412	−2.493	1.087	−0.237	0.110	–	–	0.982
	2	2.073	−2.020	0.964	−0.274	0.170	–	–	0.986
	3	1.827	−1.492	0.648	−0.316	0.232	–	–	0.985
	4	1.703	−1.384	0.698	−0.255	0.144	–	–	0.984
A4	1	1.715	−1.360	0.675	−0.224	0.100	–	–	0.997
	2	1.584	−1.130	0.567	−0.238	0.120	–	–	0.995
	3	1.601	−1.256	0.689	−0.240	0.122	–	–	0.993
	4	1.429	−0.962	0.573	−0.204	0.072	–	–	0.996
A5	1	2.245	−2.125	0.901	−0.152	−0.010[a]	–	–	0.988
	2	2.119	−1.979	0.884	−0.213	0.082	–	–	0.995
	3	1.988	−1.867	0.936	−0.131	−0.040[a]	–	–	0.991
	4	1.721	−1.362	0.673	−0.210	0.079	–	–	0.994

[a] Not significantly different from zero on a *t* test.

coefficient c_{50} was in most cases positive, but appeared often negative in group C (in 9 cases on a total of 20). Note, nevertheless, that in these cases the estimated value was not significantly different from zero, except for subject C5, session 2.

The sign requirements for the linear and non-linear damping terms were met in most cases, except for subject A1, session 1, with negative estimates for c_{01} and c_{21}, and for subject C5, session 3, with negative estimates for c_{01}, c_{21} and c_{41} (nevertheless in these two cases the estimates for c_{21} and c_{41} were not significantly different from zero). Another problem appeared with the "Rayleigh" subject C2 (session 3) with negative estimates for c_{01} and c_{03} (both significant). In all other cases, the linear damping term was negative, and at least one of the non-linear damping coefficients was positive, satisfying the basic requirements to give rise to a limit-cycle behaviour (Beek et al., 1996). It is important to note than in most cases, violations of the sign requirements were met in the presence of very weak non-linearities. The fact

Table 3
Estimates of the stiffness and damping coefficients for the individual normalised cycles of group B

Group	Session	c_{10}	c_{30}	c_{50}	c_{01}	c_{21}	c_{41}	c_{03}	r^2
B1	1	1.956	−1.604	0.690	−0.188	0.244	−0.273	–	0.997
	2	1.770	−1.435	0.725	−0.193	0.221	−0.227	–	0.996
	3	1.742	−1.207	0.485	−0.164	0.062[a]	−0.067	–	0.997
	4	1.879	−1.350	0.472	−0.113	−0.121	0.078	–	0.999
B2	1	1.777	−1.326	0.583	−0.167	0.166[a]	−0.204	–	0.986
	2	1.872	−1.554	0.722	−0.163	0.088[a]	−0.107[a]	–	0.988
	3	2.137	−2.015	0.905	−0.296	–	–	0.148	0.974
	4	1.764	−1.502	0.798	−0.251	0.332	−0.262	–	0.993
B3	1	2.284	−2.401	1.123	−0.244	0.124	–	–	0.961
	2	2.044	−1.914	0.887	0.212	0.080	–	–	0.975
	3	1.622	−1.359	0.766	−0.187	0.046[a]	–	–	0.955
	4	1.736	−1.542	0.859	−0.162	0.010[a]	–	–	0.981
B4	1	2.399	−2.454	1.076	0.213	0.076	–	–	0.995
	2	2.054	−2.043	1.046	−0.177	0.028[a]	–	–	0.993
	3	1.988	−1.835	0.867	−0.236	0.116	–	–	0.983
	4	1.919	−1.543	0.606	−0.297	0.203	–		0.996
B5	1	2.268	−2.079	0.836	−0.167	0.132	−0.167	–	0.995
	2	1.957	−1.471	0.513	−0.194	0.107	−0.074	–	0.997
	3	1.437	−0.595	0.146	−0.195	0.099	−0.055	–	0.998
	4	1.265	−0.293	0.002[a]	−0.211	0.101	−0.027	–	0.998

[a] Not significantly different from zero on a *t* test.

that our estimates were not significantly different from zero did not signify that they were equivalent to zero (an assumption leading in most cases to unrealistic models), but rather that the limit-cycle did not contain sufficient information for a valid assessment of at least some of the coefficients.

Multiple 3 (amplitude) × 4 (session) ANOVAs were performed on the individual estimates of each coefficient. These analyses revealed a significant effect of amplitude for the coefficients c_{10}, c_{30}, and c_{50} ($F_{1,12} = 14.081$, $P < 0.001$; $F_{1,12} = 22.690$, $P < 0.001$; and $F_{1,12} = 26.417$, $P < 0.001$, respectively). Post-hoc tests revealed no significant differences for the coefficients between groups A (30 cm) and B (22.5 cm), but c_{10}, c_{30} and c_{50} were significantly lower (in absolute values) for group C (15 cm) than for the two other groups. A significant effect of practice was also obtained for the three coefficients ($F_{3,36} = 18.029$, $P < 0.001$; $F_{3,36} = 16.623$, $P < 0.001$; and $F_{3,36} = 8.126$, $P < 0.001$, respectively). In all cases a strong linear trend was evident ($F_{1,12} = 29.253$, $P < 0.001$; $F_{1,12} = 26.928$, $P < 0.0001$; and $F_{1,12} = 25.628$, $P < 0.001$, respectively). Nevertheless, post-hoc tests indicated that the decrease in c_{10}, and c_{30} (in absolute values) was only significant from

Table 4
Estimates of the stiffness and damping coefficients for the individual normalised cycles of group C

Group	Session	c_{10}	c_{30}	c_{50}	c_{01}	c_{21}	c_{41}	c_{03}	r^2
C1	1	1.430	−0.476	0.016[a]	−0.158	0.004[a]	–	–	0.999
	2	1.336	−0.343	−0.032[a]	−0.207	0.073	–	–	0.998
	3	1.320	−0.354	0.000[a]	−0.215	0.086	–	–	1.000
	4	1.176	−0.188	−0.023[a]	−0.241	0.121	–	–	0.999
C2	1	1.528	−0.573	0.007[a]	−0.287	–	–	0.138	0.998
	2	1.362	−0.366	−0.026[a]	−0.180	–	–	0.025[a]	0.998
	3	1.413	−0.455	0.002[a]	−0.111	–	–	−0.048	0.999
	4	1.356	−0.373	−0.008[a]	−0.221	–	–	0.070	0.999
C3	1	1.724	−0.791	−0.006[a]	−0.191	0.045	–	–	0.994
	2	1.234	−0.281	0.012[a]	−0.251	0.138	–	–	0.998
	3	1.398	−0.442	0.010[a]	−0.193	0.054	–	–	0.999
	4	1.208	−0.218	−0.002[a]	−0.219	0.092	–	–	0.995
C4	1	1.400	−0.575	0.170	−0.204	0.110	−0.051	–	0.999
	2	1.356	−0.474	0.113	−0.147	0.002[a]	−0.019[a]	–	0.997
	3	1.294	−0.375	0.078	−0.156	0.042[a]	−0.054	–	0.999
	4	1.488	−0.762	0.290	−0.164	0.083	−0.095	–	0.999
C5	1	1.651	−0.682	−0.018[a]	−0.125	0.021[a]	−0.096	–	0.999
	2	1.495	−0.505	−0.027	−0.143	0.022	−0.056	–	
	3	1.498	−0.533	10.002[a]	−0.144	−0.002[a]	−0.022[a]	–	0.997
	4	1.508	−0.578	0.029[a]	−0.191	0.049	–	–	0.998

[a] Not significantly different from zero on a t test.

the 1st session to the 3rd, with no differences between the 2 last sessions. Post-hoc tests on c_{50} indicated only a significant decrease from session 2 to session 3. Finally, no interaction effects between practice and amplitude were found.

The statistical analysis of the individual coefficients c_{01} failed to confirm the amplitude effect suggested by the group data (see Table 1). No significant effect was obtained, neither for amplitude nor for practice. The effects of practice and amplitude on non-linear damping terms were difficult to evaluate, because the non-linear escapement functions varied strongly from subject to subject. In spite of this interindividual variation, an ANOVA was performed on the c_{12} (Van der Pol) coefficients, recognising fact that nine subjects exhibited a Van der Pol-behaviour of order two during the entire experiment (five subjects for group A, two for group B, two for group C). This analysis did not produce any significant main or interaction effect.

5. Discussion

A graphical analysis allowed us to determine the terms to be included in a minimal dynamical model of the motion of the platform of the ski-simulator produced by human subjects. When performed on group normalised cycles, this analysis revealed the presence of two non-linear Duffing stiffness terms (one cubic, the other quintic) as well as the presence of one or two non-linear damping terms from the Van der Pol series. A subsequent quantitative analysis showed that a dynamical model consisting of Van der Pol and Duffing terms adequately captured the main kinematic features of the platform motion. The identification of relevant terms by means of graphical analysis turned out to be a necessary first step before quantitative procedures such as the W-method could be applied (see also Mottet & Bootsma, 1999).

Our attempts to model individual data showed that the basic Duffing + Van der Pol model applied to most cases. Individual versions of the model varied with regard to the number of Van der Pol terms that had to be included (one or two), but in most cases the particular composition of the model appeared stable over sessions. The data of one subject, however, were better approximated by a model involving Rayleigh instead of Van der Pol terms. The particular kind of behaviour associated with Rayleigh terms was also observed in one trial of one subject of group B. These results highlight the need to construct individual-specific models, as suggested by Beek et al. (1996).

Note that, contrary to Beek et al.'s results, the individual r^2's were not markedly higher than the overall r^2. This could have been due to our data reduction method that involved the computation of averaged normalised cycles, though other explanations (e.g., task constraints) cannot be ruled out. In any event, in the present study the averaged model was representative for the behaviour of the majority of the subjects. In fact, our overall Duffing + Van der Pol model could be considered as the "true" model in that it adequately described the behaviour of most subjects.

The damping function in the derived limit-cycle model regulates the balance between energy loss and uptake. Therefore, this function is expected to be related to the forcing strategies used by subjects in sustaining the oscillations of the platform of the ski-simulator. In studying the process of learning to ski on the ski-simulator. In studying the process of learning to ski on the ski-simulator Vereijken (1991) observed that her subjects, after some practice trials, tended to force the platform just after it had passed the midpoint of the apparatus. They appeared to delay their moment of forcing, exploiting first the energy stored in the springs, and then complementing these forces with active muscular force when necessary. This strategy was adopted by all subjects in Vereijken's experiment after 8 min of cumulative practice. A consequence of such a strategy is that peak velocity should be reached each half-cycle after passing the midpoint. It is useful to note that the corresponding skewedness of the phase–plane trajectory is a specific feature of Van der Pol oscillators (Mottet & Bootsma, 1999; see Fig. 6, left).

Considering the fact that in Vereijken's experiment amplitude was left free to vary with learning, one could suppose that the adoption of this forcing strategy was related to the amplitude attained by the subjects: beyond a critical amplitude, this delayed forcing appeared as a compulsory behaviour, the only viable solution to sustain the oscillations of the platform. Viewed in this light, it is not all that surprising that the behaviour of most of our subjects was adequately captured by a limit-cycle model with one or two Van der Pol terms. It should be noticed that the (few) exceptions to this rule occurred in groups C and B, i.e., in the smaller amplitude conditions, and not all in group A, i.e., the condition with the largest amplitude.

The identified "common" forcing strategy was clearly not adopted by our "Rayleigh" subject. It must be noticed that Rayleigh and Van der Pol terms act orthogonally in phase-space (because the first is velocity-driven, and the second position-driven). As a consequence, a Rayleigh oscillator reaches its peak velocity in the first part of the half-cycle (see Fig. 6). Hence, we may suppose that our "Rayleigh" subject forced the platform before the midpoint. Vereijken (1991) observed this kind of behaviour during the very first trials of her experiment, when the subjects oscillated at very small amplitudes. The fact that our "Rayleigh" subject was dealing with the easiest

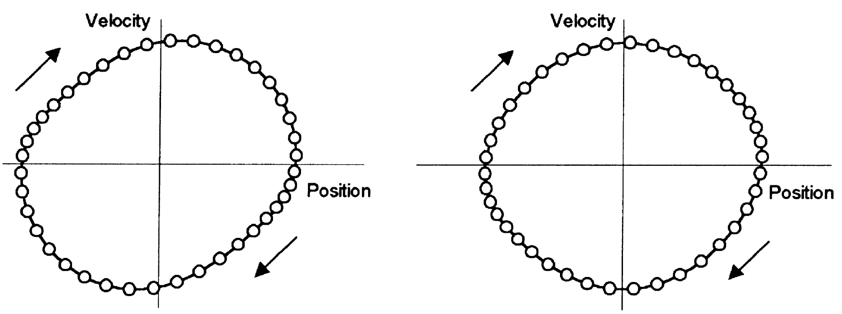

Fig. 6. Typical phase portraits, for a "Van der Pol" subjects (left, subject A2, 2nd session), and the "Rayleigh" subject (right, subject C2, 1st session). Note the contrasting skewness of the limit cycles, albeit less pronounced for the "Rayleigh" subject.

amplitude condition is consistent with our previous argument. It is surprising, however, that this subject persisted in this non-optimal behaviour during the entire experiment. Post-experimental debriefings did not provide a plausible explanation (e.g., particular previous experiences) for this observation.

The stiffness function in our model involves two non-linear Duffing terms, one cubic, the other quintic. One could suppose that this complex stiffness function reflects, at least in part, the non-linear behaviour of the simulator's springs. Vereijken (1991) performed a static stretching experiment on the springs showing that the relation between displacement (x) and resulting force (F) could be fitted by a third-order polynomial function ($F = 410.8x - 128.58x^3$, $r^2 = 0.99$, F in newtons, x in metres; see Vereijken, 1991, p. 64), suggesting that the rubber belts of the ski-apparatus could be modelled as soft springs. Nevertheless, we failed to replicate this result with our own apparatus: at least within the range of amplitudes used in the present experiment, the relation between displacement and resulting force appeared perfectly linear ($F = 1715.9x$, $r^2 = 0.99$). This suggests that the non-linear stiffness function revealed in our experiment cannot be exclusively explained by the mechanical properties of the springs, but rather reflects the behaviour of the subject-apparatus system. This point of view is consistent with the progressive linearisation, with practice, of the stiffness function.

The negative cubic term indicates a local slowing down of the frequency in the neighbourhood of the maximal excursion. Mottet and Bootsma (1999) interpreted such local behaviour in Fitts' tasks as a means to increase the dwell time near the targets for high levels of task difficulty. In the present experiment, this softening component could be related to the postural adjustments necessary to stabilise the reversal point of the movement. This interpretation is supported by the fact that the magnitude of the non-linear stiffness components is related to the amplitude constraints, and tends to decrease with practice. An analysis of the segmental co-ordination is required to further develop this point. Nevertheless, the learning stages on the ski-simulator proposed by Vereijken et al. (1997) suggest that subjects progressively learn to exploit the reactive forces of the system. Viewed against this background, the observed progressive linearisation of the stiffness function is not an unexpected result.

Our results confirm the idea, notably defended by Beek et al. (1995a,b) and Mottet and Bootsma (1999), that non-linear stiffness is an important component in biological movements (remember that a main assumption in Kay et al.'s (1987) approach was the linearity of the stiffness function). Such a combination of cubic and quintic Duffing terms was extensively studied by Gonzales and Piro (1987), and used by Schöner (1990) in his attempt to construct a dynamical model for discrete movements. Mottet and Bootsma (1999) claimed that the inclusion of a quintic term is general required in such equations, because a model containing only a cubic component diverges to infinity when position reaches a root of the stiffness function. When a quintic term is added, the stiffness function exhibits local minima in the neighbourhood of the reversal points, allowing a stable behaviour both within and outside the inter-root interval.

Visual inspection of Table 1 suggests an increase of the contribution of linear and non-linear dampings as the required amplitude increases. This observation was not supported by the analysis of variance performed on the individual coefficients. Nevertheless, Beek et al. (1995a,b) recognised that their W-method was less powerful with regard to the assessment of dissipative components. Therefore, our estimates based on individual data must be considered with caution. Note that an increase of the contribution of the damping function leads to an enhancement of the stability of the system (Mottet & Bootsma, 1999): graphical simulations clearly show that an increase of the coefficients of the damping terms lead to a more stable limit-cycle with a shorter relaxation time. Such trend could underlie the observed effect of required amplitude on frequency variability.

An important finding in the present study is the qualitative stability of individual models across practice sessions. Generally, a unique model was shown to provide an accurate account of individual data from the first to the last session. Practice led to quantitative changed of the coefficients of the model, but the model itself remained qualitatively unchanged. This suggests that, at least within the window explored in the present experiment, practice results in continuous changes in co-ordination, rather than in abrupt transitions. This conclusion is not in accordance with the observations of Vereijken et al. (1997), who described skill acquisition on the ski-simulator as the succession of three qualitatively distinct stages, characterised by different mechanical models. Nevertheless, our subjects, while naive on the ski-simulator appeared to benefit of their previous ski experiences. They were able from the very first trials to reach the required amplitudes, even in the A (30 cm) and B (22.5 cm) groups, when the average initial amplitudes reported in Vereijken's experiments were around 10 cm (Vereijken, 1991). This suggests that our subjects, from the beginning of the experiment, were beyond the first stage identified by Vereijken (the "balancing pendulum").

Only two exceptions appeared to this qualitative stability of individual models: (1) subject B2 presented a transitory shift, in session 3, from the Van der Pol model to the Rayleigh model, and (2) data from the last session of subject C5 were better accounted for by the one-term Van der Pol model Eq. (3), whereas the first 3 sessions were accurately described by the alternative Van der Pol model Eq. (4). The final shift of subject C5 could be interpreted as a logical evolution of skill with practice, as Eq. (3) corresponds to the most common model, at group level as well as individual level. The transitory shift of subject B2 is more difficult to explain. Further analyses (considering other data samples from this particular session) could indicate whether this shift was due to a deliberate change in strategy (i.e., an active exploration of alternative solutions) or to contingent factors.

6. Conclusion

The present article illustrates that the graphical and numerical methods proposed by Beek and Beek (1988) constitute valuable tools for constructing dynamical models of biological rhythmic movements. When applied to averaged normalised cycles, eliminating the effect of random noise in the experimental data, these methods produce suitable models, both from a qualitative (sign constraints) and from a quantitative point of view (fitting accuracy). Following Mottet and Bootsma (1999), we emphasise the necessity of a preliminary selection of relevant terms before applying the W-method. One could note, however, that our results highlight the difficulty to obtain reliable estimates of the damping coefficients from phase-plane data. This observation appears particularly relevant concerning individual-specific modelling, and needs further methodological considerations. At a more general level, and in accordance with Beek et al.'s (1996) conclusions, our results confirm two important points:

1. the importance of stiffness non-linearities in biological movements;
2. the necessity to construct individual-specific models to do justice to the inter-individual variations observed in the kinematics.

These dynamical models offer a very rich description of rhythmical movements. Our results highlight role of tasks constraints, which appeared to determine the nature of the model as well as the relative importance of each term. Additionally, our results suggest that practice leads to a progressive tuning of the dynamics of the movement, that may be characterised by a gradual linearization of the underlying model. Nevertheless, this experiment was not designed to study the entire process of learning. Further research is needed to assess the capability of dynamical models, such as the limit-cycle models derived in the present study, to account for the qualitative changes which punctuate the learning of a new skill (i.e., phase transitions). Such models could provide valuable order parameters for the study of complex skills acquisition.

A final comment is perhaps in place to address an often raised question: Does this kind of modelling effort represent anything more than a sophisticated form of curve fitting? Does it allow to advance our understanding of how movements are learned and controlled? Our goal cannot be reduced to identifying the mathematical equations that perfectly fit our experimental data. The class of models we try to construct possess dynamical properties, such as stability, regime and bifurcation behaviour (Gonzalez & Piro, 1987; Holmes & Rand, 1980; Mottet & Bootsma, 1999). The basic assumption of this approach is that the system *exploits* these properties (i.e., the limit-cycle behaviour) to produce rhythmic movements. Our goal is less to describe the movement than to identify the attractor underlying it. The finding of the present study that this attractor remains qualitatively unchanged over five consecutive sessions of practice clearly goes beyond curve fitting.

References

Beek, P. J., & Beek, W. J. (1988). Tools for constructing dynamical models of rhythmic movement. *Human Movement Science*, *7*, 301–342.

Beek, P. J., Peper, C. E., & Stegeman, D. F. (1995a). Dynamical models of movement coordination. *Human Movement Science*, *14*, 573–608.

Beek, P. J., Schmidt, R. C., Morris, A. W., Sim, M. Y., & Turvey, M. T. (1995b). Linear and non-linear stiffness and friction in biological rhythmic movements. *Biological Cybernetics*, *73*, 499–507.

Beek, P. J., Rikkert, W. E. I., & van Wieringen, P. C. W. (1996). Limit-cycle properties of rhythmic forearm movement. *Journal of Experimental Psychology: Human Perception and Performance*, *22*, 1077–1093.

Delignières, D., Nourrit, D., Lauriot, B., & Cadjee, I. (1997). L'acquisition des habiletés motrices complexes. Paper presented to the Seventh International Congress of ACAPS, Marseille, France.

Brinker, B. P. L. M., & van Hekken, M. F. (1982). The analysis of slalom-ski type movements using ski-simulator apparatus. *Human Movement Science*, *1*, 91–108.

den Brinker, B, Stäbler, J., Whiting, H. T. A., & Wieringen, P. C. W. (1986). The effect of manipulating knowledge of results on the learning of slalom-type ski movements. *Ergonomics*, *29*, 31–40.

Durand, M., Geoffroi, V., Varray, A., & Préfaut, C. (1994). Study of the energy correlates in the learning of a complex self-paced cyclical skill. *Human Movement Science*, *13*, 785–799.

Gonzalez, D. L., & Piro, O. (1987). Global bifurcations and phase portrait of an analytically solvable non-linear oscillator: Relaxation oscillations and saddle-node collision. *Physical Review A*, *36*, 4402–4410.

Guiard, Y. (1993). On Fitt's and Hooke's laws: Simple harmonic movement in upper-limb cyclical aiming. *Acta Psychologica*, *82*, 139–159.

Haken, H. (1983). *Advance synergetics*. Heidelberg: Springer.

Haken, H., Kelso, J. A. S., & Bunz, H. (1985). A theoretical model of phase transition in human hand movement. *Biological Cybernetics*, *51*, 347–356.

Holmes, P., & Rand, D. (1980). Phase portraits and bifurcations of the non-linear oscillator: $\ddot{x} + (\alpha + \gamma x^2)\dot{x} + \beta x + \delta x^3 = 0$. *International Journal of Non-Linear Mechanics*, *15*, 449–458.

Kay, B. A., Saltzman, E. L., Kelso, J. A. S., & Schöner, G. (1987). Space–time behaviour of single and bimanual rhythmical movements: Data and limit-cycle model. *Journal of Experimental Psychology: Human Perception and Performance*, *13*, 178–192.

Mottet, D., & Bootsma, R. J. (1999). The dynamics of goal-directed rhythmical aiming. *Biological Cybernetics*, *80*, 235–245.

Newell, K. M. (1980). The speed accuracy paradox in movement control errors of time and space. In G. E. Stelmach & J. Requin, *Tutorials in motor behaviour* (pp. 501–510). Amsterdam: North-Holland.

Schöner, G. (1990). A dynamic theory of coordination of discrete movement. *Biological Cybernetics*, *63*, 257–270.

van Emmerik, R. E. A., den Brinker, B. P. L. M., Vereijken, B., & Whiting, H. T. A. (1989). Preferred tempo in the learning of a gross cyclical action. *The Quarterly Journal of Experimental Psychology: Human Experimental Psychology*, *41A*, 251–262.

Vereijken, B. (1991). The dynamics of skill acquisition. Ph.D. thesis. Vrije Universiteit, Amsterdam. Meppel: Krips Repro.

Vereijken, B., & Whiting, H. T. A. (1990). In defence of discovery learning. *Canadian Journal of Sport Science*, *15*, 99–106.

Vereijken, B., Whiting, H. T. A., & Beek, W. J. (1992a). A dynamical systems approach to skill acquisition. *The Quarterly Journal of Experimental Psychology*, *45A*, 323–344.

Vereijken, B., van Emmerik, R. E. A., Whiting, H. T. A., & Newell, K. M. (1992b). Free(z)ing degrees of freedom in skill acquisition. *Journal of Motor Behaviour*, *24*, 133–142.

Vereijken, B., van Emmerik, R. E. A., Bongaardt, R., Beek, W. J., & Newell, K. M. (1997). Changing coordinative structures in complex skill acquisition. *Human Movement Science*, *16*, 823–844.

Whiting, H. T. A., Vogt, S., & Vereijken, B. (1992). Human skill and motor control: Some aspects of the motor control–motor learning relation. In J. J. Summers, *Approaches to the study of motor control and learning* (pp. 81–111). Amsterdam: North-Holland.

Wulf, G., Höß, M., & Prinz, W. (1998). Instructions for motor learning: Differential effects of internal versus external focus of attention. *Journal of Motor Behavior*, *30*, 169–179.

Wulf, G., & Weigelt, C. (1997). Instructions about physical principles in learning a complex motor skill: To tell or not to tell. *Research Quarterly for Exercice and Sport*, *68*, 362–367.

Zaal, F. T. J. M., Bootsma, R. J., & van Wieringen, P. C. W. (1999). Dynamics of reaching for stationary and moving objects: Data and model. *Journal of Experimental Psychology: Human Perception and Performance*, *25*, 149–161.